Possible solution of the cosmological constant problem in the framework of lattice quantum gravity

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It is shown that in the theory of discrete quantum gravity the cosmological constant problem can be solved due to the phenomena of elliptic operators spectrum "loosening" and universe inflation.

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1. Let's outline shortly the cosmological constant problem (see, for example, [1]).

Consider Einstein equation with Λ-term ($\hbar = c = 1$):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} + \Lambda\, g_{\mu\nu}\,, \tag{1}$$

$$G \sim l_P^2 \sim 2,5 \cdot 10^{-66}\, cm^2\,. \tag{2}$$

According to today experimental data

$$T_{\mu\nu} \sim 10^8 cm^{-4} \longrightarrow 8\pi\, G\, T_{\mu\nu} \sim 5 \cdot 10^{-57} cm^{-2}\,, \tag{3}$$

$$\Lambda \sim 10^{-56} cm^{-2}\,. \tag{4}$$

Thus, if Einstein equation (1) is applied to the today dynamics of universe, the quantities in its right hand side are of the same order indicated in (3) and (4).

On the other hand, any elementary estimation of the right hand side of Eq. (1) in the framework of canonical quantum field theory gives extremely large value in comparison with the experimental data (3) and (4). Indeed, the vacuum expectation value of the energy-momentum tensor in free quantum field theory has the order

$$\langle T_{\mu\nu} \rangle_0 \sim \int_{|\mathbf{k}|<k_{max}} \frac{\mathrm{d}^{(3)}\, k}{(2\pi)^3} \left(\frac{k_\mu k_\nu}{k^0} \bigg|_{k^0 = |\,\mathbf{k}\,|} \right)\,. \tag{5}$$

Here k^μ is the 4-momentum the corresponding mode. If in the integral (5) $k_{max} \sim l_P^{-1}$ (Planck scale), then it follows from (5) and (2)

$$8\pi G \langle T_{\mu\nu} \rangle_0 \sim l_P^{-2} \sim 10^{66}\, cm^{-2}\,. \tag{6}$$

It is clear that the interaction of fields doesn't changes qualitatively the estimation (6) which in any case is incompatible with the experimental estimations (3), (4).

It is well known that the solution of the outlined problem is absent at present [1], though some interesting ideas have been appeared lately (see, for example [2], [3], [4]). In this letter I do try to present the qualitative estimations showing the compatibility of discrete quantum gravity in quasi-classical state with the cosmological experimental data (3) and (4). This means that in the considered theory the vacuum expectation value of the energy-momentum tensor becomes enough small due to (i) the phenomenon of "loosening" of elliptic operators spectrum and (ii) inflation of universe. Moreover, the quantum degrees of freedom exist right up to Planck scale. The estimations are based on the results obtained in the papers [4], [5], [6].

2. It is necessary to write out some formulae from the work [6]. Here the notations are completely identical to that in [6].

Let $\mathfrak{K}$ be a 4-dimensional simplicial complex such that the 3-dimensional complex $\mathfrak{S} = \partial\,\mathfrak{K}$ has the topology of 3-sphere S^3. To each vertex $a_i \in \mathfrak{K}$, the Dirac spinors ψ_i and $\overline{\psi}_i$ belonging to the complex Grassman algebra are assigned. To each oriented edge $a_i a_j \in \mathfrak{K}$, an element of the group $Spin(4)$

$$\Omega_{ij} = \Omega_{ji}^{-1} = \exp\left(\frac{1}{2} \omega_{ij}^{ab} \sigma^{ab} \right)\,, \quad \sigma^{ab} = \frac{1}{4}[\gamma^a,\, \gamma^b]\,, \tag{7}$$

and also an element $\hat{e}_{ij} \equiv e_{ij}^a \gamma^a$, such that

$$\hat{e}_{ij} = -\Omega_{ij} \hat{e}_{ji} \Omega_{ij}^{-1}, \quad -\infty < e_{ij}^a < \infty, \tag{8}$$

are assigned. The notations $\overline{\psi}_{Ai}$, ψ_{Ai}, $\hat{e}_{Aij}$, Ω_{Aij} and so on indicate that the edge $X_{ij}^A = a_i a_j$ belongs to 4-simplex with index A. Let a_{Ai}, a_{Aj}, a_{Ak}, a_{Al}, and a_{Am} be all five vertices of a 4-simplex with index A and $\varepsilon_{Aijklm} = \pm 1$ depending on whether the order of vertices $a_{Ai}\, a_{Aj}\, a_{Ak}\, a_{Al}\, a_{Am}$ defines the positive or negative orientation of this 4-simplex. The Euclidean action of the theory has the form

$$\begin{gathered} I = \tfrac{1}{5\times 24} \textstyle\sum_A \sum_{i,j,k,l,m} \varepsilon_{Aijklm}\, \mathrm{tr}\, \gamma^5 \times \\ \times \left\{ -\tfrac{1}{2\, l_P^2} \Omega_{Ami} \Omega_{Aij} \Omega_{Ajm} \hat{e}_{Amk} \hat{e}_{Aml} + \right. \\ \left. + \tfrac{i}{48} \gamma^a \left(\overline{\psi}_{Ai} \gamma^a \Omega_{Aij} \psi_{Aj} - \overline{\psi}_{Aj} \Omega_{Aji} \gamma^a \psi_{Ai} \right) \hat{e}_{Amj} \hat{e}_{Amk} \hat{e}_{Aml} \right\}. \end{gathered} \tag{9}$$

The oriented volume of a 4-simplex with vertexes a_{Ai}, a_{Aj}, a_{Ak}, a_{Al}, and a_{Am} is equal to

$$V_A = \frac{1}{4!} \frac{1}{5!} \sum_{i,j,k,l,m} \varepsilon_{A\, ijklm}\, \varepsilon^{abcd}\, e_{A\, mi}^a e_{A\, mj}^b e_{A\, mk}^c e_{A\, ml}^d\,. \tag{10}$$

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The quantity $l_{ij}^2 = \sum_{a=1}^4 (e_{ij}^a)^2$ is interpreted as the square of the length of the edge $a_i a_j$.

The partition function Z for a discrete Euclidean gravity [8] is defined as follows:

$$Z = \text{const} \cdot \left(\prod_{\mathcal{E}} \int \mathrm{d}\,\Omega_{\mathcal{E}} \int \mathrm{d}\, e_{\mathcal{E}} \right) \times \\ \times \big(\prod_{\mathcal{V}} \mathrm{d}\, \overline{\psi}_{\mathcal{V}}\ \mathrm{d}\, \psi_{\mathcal{V}} \big) \exp \big(-I \big) . \tag{11}$$

Here, the vertices and edges are enumerated by indices $\mathcal{V}$ and $\mathcal{E}$, and the corresponding variables are denoted by $\psi_{\mathcal{V}}$, $\Omega_{\mathcal{E}}$, respectively, $\mathrm{d}\,\Omega_{\mathcal{E}}$ is the Haar measure on the group Spin(4), and

$$\mathrm{d}\, e_{\mathcal{E}} \equiv \prod_a \mathrm{d}\, e_{\mathcal{E}}^a , \quad \mathrm{d}\, \overline{\psi}_{\mathcal{V}}\ \mathrm{d}\, \psi_{\mathcal{V}} \equiv \prod_\nu \mathrm{d}\, \overline{\psi}_{\mathcal{V}\nu}\ \mathrm{d}\, \psi_{\mathcal{V}\nu} . \tag{12}$$

The index ν enumerates individual components of the spinors $\psi_{\mathcal{V}}$ and $\overline{\psi}_{\mathcal{V}}$.

Let's denote by $\mathcal{X}$ a 4-dimensional smooth manifold with topology of the complex $\mathfrak{K}$. Consider a set of maps $\{g\}$ from geometrical realization of the complex $\mathfrak{K}$ onto manifold $\mathcal{X}$ which are not necessarily one-one maps. For a given local coordinates x^μ, $\mu = 1, 2, 3, 4$ a map g defines the coordinates of images of vertexes $a_{A\,i}$: $x^\mu_{g(A\,i)} \equiv g^\mu(a_{A\,i})$. Define the four differentials

$$\mathrm{d}\, x^\mu_{g(A\,ji)} \equiv x^\mu_{g(A\,i)} - x^\mu_{g(A\,j)} , \quad i \neq j , \quad i = 1, \ldots, 4. \tag{13}$$

Suppose the smooth fields $\omega_\mu^{ab}(x), e_\mu^a(x), \overline{\psi}(x), \psi(x)$ are defined on the manifold $\mathcal{X}$. Then we can define the discrete lattice variables according to the relations

$$\omega_\mu^{ab}(x_{g(A\,m)})\, \mathrm{d}\, x^\mu_{g(A\,mi)} = \omega^{ab}_{A\,mi}, \\ e_\mu^a(x_{g(A\,m)})\, \mathrm{d}\, x^\mu_{g(A\,mi)} = e^a_{A\,mi}, \quad \psi(x_{g(A\,i)}) = \psi_{A\,i}. \tag{14}$$

On the contrary, the discrete lattice variables in the right hand sides of Eqs. (14) define the values of the fields on the images of vertexes of the complex. It is clear [6] that for the discrete lattice variables which change enough smoothly along the complex we obtain the enough smooth fields. Moreover, in this case the discrete action (10) transforms to the well known continuum action

$$I = \int \varepsilon_{abcd} \left\{ -\tfrac{1}{l_P^2} R^{ab} \wedge e^c \wedge e^d + \right. \\ \left. + \tfrac{i}{12} \left(\overline{\psi}\, \gamma^a\, \mathcal{D}_\mu \psi - \mathcal{D}_\mu \overline{\psi}\, \gamma^a\, \psi \right) \mathrm{d}\, x^\mu \wedge e^b \wedge e^c \wedge e^d \right\} , \\ e^a = e^a_\mu\, \mathrm{d}\, x^\mu, \qquad \omega^{ab} = \omega^{ab}_\mu\, \mathrm{d}\, x^\mu, \\ \mathrm{d}\, \omega^{ab} + \omega^a_c \wedge \omega^{cb} = \tfrac{1}{2} R^{ab}. \tag{15}$$

I emphasize that we obtain the action (16) only if the lowest derivatives of the fields are taken into account. It is important that in this case the information on the structure of the complex is lost. This is incorrectly if the highest derivatives of the fields are also taken into account. In the work [6] the arguments have been given that the quasi-classical phase at the same time is the macroscopic continuum phase with long correlations and hence, also, the phase in which the highest derivatives of the fields can be ignored. In this phase the partition function is saturated by normal (smooth) modes but not by anomal modes responsible for Wilson state doubling [5].

It is important that in the quasi-classical phase the universe wave function does not depend on the discrete variables $e^a_{A\,ij}$ in a wide diapason. This statement is true in the same context as the highest derivatives of the fields can be ignored. Indeed, in the quasi-classical phase the action (16) as well as the universe wave function depend on the fields $\omega_\mu^{ab}(x), e_\mu^a(x), \overline{\psi}(x), \psi(x)$ which are present in the left hand side of Eqs. (14). Thus, fixing these fields and varying the maps g or, equivalently, the images of vertexes $x^\mu_{g(A\,i)}$ and, hence, the differentials $\mathrm{d}\, x^\mu_{g(A\,ji)}$, one can vary the discrete variables $e^a_{A\,mi}$ in the right hand side of Eqs. (14) in a wide region.

To clarify the situation, let's consider the case when geometry of the space-time is flat or almost flat. In the flat case one can take

$$\omega_{ij}^{ab} = 0 , \quad \left(e^a_{ij} + e^a_{jk} + \ldots + e^a_{li} \right) = 0 . \tag{16}$$

Here, the sum in the parentheses is taken over any closed path formed by 1-simplices. Equations (16) indicates that the curvature and torsion are equal to zero. Thus, geometrical realization of the complex $\mathfrak{K}$ is in the four-dimensional Euclidean space, e^a_{ij} being the components of the vector in a certain orthogonal basis in this space, and if R^a_i is the radius-vector of vertex a_i, then $e^a_{ij} = R^a_j - R^a_i$. In this case one can take $e^a_\mu(x) = \delta^a_\mu$. It is evident that Eqs. (16) are the only restrictions for the variables e^a_{ij}.

3. Now we are able to study the problem of spectrum loosening. Here the spectrums of elliptic operators on the subcomplex $\partial\,\mathfrak{K}$ are studied. Note that although even in the simplest case the subcomplex $\partial\,\mathfrak{K}$ has the topology of 3-sphere S^3, here one can consider enough large subcomplexes of $\partial\,\mathfrak{K}$ as flat one and keep in mind Eqs. (16).

Firstly, I write out the trivial formula for the volume in momentum space occupied by all modes of scalar field defined on the vertexes of periodic cubic lattice with the total number of vertexes N and volume V Here the spectrum of elliptic operators on the subcomplex $\partial\,\mathfrak{K}$:

$$\Omega = (2\pi)^3 \frac{N}{V} . \tag{17}$$

The estimation (17) remains true for all fields with the spin of the order of one; moreover, this estimation remains qualitatively true for the case of irregular lattice when the spacings between neighbor vertexes are commensurable. I say that in this case the modes are densely packed in momentum space.

Now let's pass to the spectrum "loosening" problem in the discrete quantum gravity.

The subsequent estimations in this Subsection are made for the intermediate regime from confinement phase to the quasi-classical phase. In [6] the arguments are given that in this regime the fields at nearest regions of space volume are correlated weakly. It is natural to assume that the same conclusion remains true at initial times in quasi-classical phase. Therefore let us divide the macroscopic volume V with the total number of degrees of freedom (or the number of modes which in one's turn is of the order of the number of vertexes of the complex) N into $\mathcal{N}$ subvolumes v_i in each of which n_i degrees of freedom is contained. Thus

$$\sum_{i=1}^{\mathcal{N}} n_i = N\,, \qquad \sum_{i=1}^{\mathcal{N}} v_i = V\,, \tag{18}$$

and

$$\omega_i = (2\pi)^3 \frac{n_i}{v_i} \tag{19}$$

is the minimal possible volume in momentum space occupied by n_i modes placed in the volume v_i. Now instead of the quantity (17) one must consider the following quantity

$$\tilde{\Omega} = \frac{(2\pi)^3}{\mathcal{N}} \sum_{i=1}^{\mathcal{N}} \frac{n_i}{v_i}\,. \tag{20}$$

Indeed, the minimum of quantity (20) subjected to the constraints (18) is equal to (17).

Since in the considered theory the volumes v_i are variable quantities, one must introduce the measure on the manifold of volumes $\{v_i\}$. In [6] the measure

$$\mathrm{d}\,\mu = \frac{(\mathcal{N}-1)!}{V^{\mathcal{N}-1}} \delta\left(V - \sum_{i=1}^{\mathcal{N}} v_i\right) \prod_{i=1}^{\mathcal{N}} \mathrm{d}\,v_i\,, \quad v_i > 0\,,$$
$$\int \mathrm{d}\,\mu = 1 \tag{21}$$

was suggested.

To justify the measure (21), I note that the volumes and the forms of elementary cells of the space are arbitrary and mutually independent, and the wave function of universe depend weakly on these values in wide diapason in the quasi-classical phase of the theory (see the previous Subsection). The volumes of elementary cells are determined according to Eq. (10) only by 1-forms e^a_{ij} which change independently in integral (11), and the action under integral for partition function depends weakly (does not depend at all on the discrete variables satisfying Eqs. (16) in the long-wave limit) on the variables e^a_{ij} in the quasi-classical phase. Moreover, the measure in integral (11) is proportional to the product of all differentials $\mathrm{d}\,e^a_{ij}$, which in one's turn is proportional to the product $\prod_{i=1}^{\mathcal{N}} \mathrm{d}\,v_i$. So we see that the simplest measure (21) is valid for averaging.

Hence, instead of (20) the more physically sensible quantity is

$$\langle \tilde{\Omega}\,\rangle \equiv \int \tilde{\Omega}\,\mathrm{d}\,\mu = (2\pi)^3 \frac{\mathcal{N}-1}{V\mathcal{N}} \sum_{i=1}^{\mathcal{N}} n_i \int_{v_i \ll V} \frac{\mathrm{d}\,v_i}{v_i} =$$
$$= (2\pi)^3 \frac{N}{V} \int_{v_i \ll V} \frac{\mathrm{d}\,v_i}{v_i}\,. \tag{22}$$

The last equality follows from the first constraint in (18) and the relation $\mathcal{N} \gg 1$.

The comparison of Eqs. (17) and (22) shows that the phenomenon of essential expansion of the momentum space volume occupied by quantum field modes arises as the consequence of the dynamics of the system. This expansion factor is of the order of

$$\varkappa_1 \sim \int_{v_i \ll V} \frac{\mathrm{d}\,v_i}{v_i} = 3 \ln \frac{a_1}{a_0} = 3 \ln \xi_0\,. \tag{23}$$

Here a_0 is some minimal dimension of the theory and $a_1 \ll V^{1/3}$. It seems that $a_0 \gg l_P$, since only at $|e^a_{ij}| \gg l_P$ the quasi-classical phase can exist (see [6]).

Now it is necessary to take into account that instead of quantities (n_i/v_i) in Eq. (20) one must use the averaged (over shortest wavelength fluctuations) quantities $\langle (n_i/v_i) \rangle$. Thus, using the obtained estimation (23) we elaborate a kind of renormalization group describing loosening of mode packing. Let J be the number of steps of renormalization group and

$$\xi_j = \frac{a_{j+1}}{a_j} = \xi \gg 1\,, \quad j = 1, \ldots, J\,, \tag{24}$$

$a_{J+1} = a_{max} \lll a$, and a is the universe radius. Thus $\xi^J = \xi_1 \xi_2 \ldots \xi_J = a_{max}/a_0$. For rough estimation let us take

$$J = \frac{1}{\lambda} \ln \frac{a_{max}}{a_0} \gg 1\,, \quad \ln \xi = \lambda \gg 1\,. \tag{25}$$

Using Eqs. (23)–(25) it is easy to see that the expansion factor of momentum space volume occupied by modes after J steps is

$$\varkappa_J = \prod_{j=1}^{J} (3 \ln \xi_j) = (3 \ln \xi)^J = \left(\frac{a_{max}}{a_0}\right)^{(\ln 3\lambda)/\lambda}\,. \tag{26}$$

The value of the right hand side of Eq. (26) can bee very large (many orders) in magnitude. This phenomenon is called here as "spectrum loosening". It seems that the effect of spectrum loosening and translational invariance are compatible on the breathing lattice.

It follows from the presented analysis, that the continuum quantum gravity arising from the discrete quantum gravity (if it exists) possess very unusual properties. For example, let's try to estimate the contribution to the cosmological constant due to the quantum field fluctuations in the framework of presented here theory taking into account the estimation (26). We shall see that following this path one can solve the problem of a large value of cosmological constant. Indeed, in the elaborated here theory the estimation (6) should be corrected by the factor $\varkappa_J^{-1} \lll 1$ for the reason of noncompact packing of the field modes in momentum space! Thus, instead of the estimation (6) now we have the following one

$$\Lambda_{eff} \sim \left(\frac{a_0}{a_{max}}\right)^{(\ln 3\lambda)/\lambda} l_P^{-2} \lll l_P^{-2} . \tag{27}$$

So the effective cosmological constant can be made enough small.

4. In [6] the arguments have been given that the quasi-classical phase of the theory does actually exist. Here in the Subsection 3 it is shown that the properties of such theory are very unusual for the reason of "spectrum loosening" phenomenon at the beginning of universe inflation. Now the question arises: what looks like such unusual continuum quantum theory of gravity? In this Subsection I try to describe phenomenologically a variant of continuum quantum theory of gravity with the necessary properties. Below under the dynamic system the continuum quantum theory of gravity is meant. Here the results of the work [4] are used. To quantize the general covariant theories I follow the well known Dirac quantization procedure. For clearness it is convenient to formulate the main assumptions in the form of axioms.

Axiom 1. *All states of the theory having physical sense are obtained from the ground state* $|\,0\,\rangle$ *using the creation operators* $A_n^\dagger$:

$$\begin{gathered} |\,n_1; \ldots ; n_s\,\rangle = A_{n_1}^\dagger \cdot \ldots \cdot A_{n_s}^\dagger \,|\,0\,\rangle \;, \\ A_n \,|\,0\,\rangle = 0 \,. \end{gathered} \tag{28}$$

States (28) form an orthogonal basis of the space of physical states of the theory.

Here the operators $A_n^\dagger$ and their conjugates A_n are the generators of bosonic and fermionic Heisenberg algebra. For the case of compact spaces which is interesting for us, the index n belongs to a discrete finite-dimensional lattice.

Since states (28) are physical, they satisfy the relations

$$\mathcal{H}_T \,|\,n_1; \ldots ; n_s\,\rangle = 0 \;, \tag{29}$$

where $\mathcal{H}_T$ is the complete Hamiltonian of the theory. We assume that $\mathcal{H}_T = \sum_\Xi v_\Xi \chi_\Xi$, where $\{\chi_\Xi\}$ is the complete set of the first class constraints and $\{v_\Xi\}$ is an arbitrary set of Lagrange multipliers.

Equations (28) and (29) are compatible if and only if the following relations are valid:

$$\begin{gathered} [A_n,\, \mathcal{H}_T] = \sum_{\Xi,\,\Pi} r_{n\,\Xi\,\Pi} v_\Xi\, \chi_\Pi \longleftrightarrow \\ \longleftrightarrow [A_n^\dagger,\, \mathcal{H}_T] = -\sum_{\Xi,\,\Pi} \chi_\Pi\, v_\Xi^* r_{n\,\Xi\,\Pi}^\dagger \,. \end{gathered} \tag{30}$$

Let an arbitrary field (or more general operator) $\Psi(x)$ be represented as a normal ordered power series in operators $(A_n^\dagger,\, A_n)$ (here the index n is fixed):

$$\Psi = \Psi' + \psi_n^{(+)} A_n + A_n^\dagger \psi_n^{(-)} \,. \tag{31}$$

Here $\psi_n^{(\pm)}$ are treated as the wave functions of the corresponding states. By definition, here the operator Ψ' does not depend on the operators $(A_n^\dagger,\, A_n)$:

$$[\Psi',\, A_n^\dagger] = [\Psi',\, A_n] = 0 \,. \tag{32}$$

It follows from Eqs. (30)–(32) that

$$\begin{gathered} [\Psi,\, \mathcal{H}_T] = [\Psi',\, \mathcal{H}_T'] + \\ + \sum_\Xi (q_\Xi \chi_\Xi + \chi_\Xi \tilde{q}_\Xi) + (p_n A_n + A_n^\dagger \tilde{p}_n) \,. \end{gathered} \tag{33}$$

Here the total Hamiltonian $\mathcal{H}_T$ is represented according to (31), so that $\mathcal{H}_T'$ does not depend on the operators $(A_n^\dagger,\, A_n)$.

Now let's impose an additional pair of second class constraints

$$A_n = 0 \,, \qquad A_n^\dagger = 0 \,. \tag{34}$$

By definition, under the constraints (34) any operator Ψ is reduced to the operator Ψ' in (31). For any operators Ψ, Φ the Dirac bracket arising under the constraints (34) is defined according to the following equality:

$$[\Psi,\, \Phi]^* \equiv [\Psi',\, \Phi'] \,. \tag{35}$$

The remarkable property of the considered theory is the fact that

$$[\Psi,\, \mathcal{H}_T]^* \approx [\Psi,\, \mathcal{H}_T] \,. \tag{36}$$

Here the approximate equality means that after the imposition of all first and second class constraints the operators in the both sides of Eq. (36) coincide, that is the weak equality (36) reduces to the strong one. Relation (36) follows immediately from Eqs. (33) and (35). Eq. (36) means that the Heisenberg equation

$$i\dot{\Psi} = [\Psi,\, \mathcal{H}_T]^* \tag{37}$$

for any field in reduced theory coincides weakly with corresponding Heisenberg equation in nonreduced theory. Evidently, this remarkable conclusion retains true under imposition of any number of pairs of the second class constraints of the type (34). Thus, it is naturally to accept

Axiom 2. *In the case of compact space the index n in axiom 1 runs a finite set of indexes: $n = 1, \ldots, N$. Moreover, it is assumed that the packing of modes in momentum space is essentially noncompact.*

Axiom 3. *The equations of motion and the constraints for the physical fields have the same form (up to the arrangement of the operators) as the corresponding classical equations and constraints.*

The axiom 2 states not only ultraviolet regularization of the theory but also the "spectrum loosening" phenomenon. The axiom 3 is a consequence of Eqs. (36) and (37).

In considered theory the totality of equations of motion and constraints include Einstein equations and matter field equations of motion. Further for brevity I shall call all these equations as equations of motion. To obtain the solutions in such theory one must substitute to the equations of motion the fields decomposed according to

$$\Psi(x) = \Psi'(x) + \sum_{n=1}^{N} \left\{ \psi_n^{(+)}(x) A_n + A_n^\dagger \psi_n^{(-)}(x) \right\} . \quad (38)$$

By definition only the wave functions $\{\psi_n^{(\pm)}\}$ are decomposed in series of operators $\{A_n^\dagger, A_n\}$ but not the field Ψ'. So the equations of motion become the series in the powers of operators $\{A_n^\dagger, A_n\}$; evidently, the c-numerical coefficients at different powers of operators $\{A_n^\dagger, A_n\}$ are equal to zero separately. Thus the chain of c-numerical differential equation arises, the zeroth approximation of which is the classical Einstein equation. It is important that the equations of motions are general covariant. This is the consequence of axiom 3 and the fact that the corresponding classical equations are general covariant.

I call the quantization of gravity stated by axioms 1–3 as dynamic quantization [4], [7].

5. Now, using the aforesaid, let's give some general estimations.

For simplicity let's consider the contribution to the cosmological constant due to Dirac sea. The contribution to the energy-momentum tensor from the massless Dirac field on the mass shell has the form

$$T_{\psi\,\mu\nu} = \frac{i}{2}\Big(\overline{\psi}\,\gamma^a\, e_{a(\mu} D_{\nu)} \psi - e_{a(\mu} \overline{D_{\nu)}\psi}\,\gamma^a\,\psi\Big), \quad (39)$$

and according to axioms 1 and 2 the Dirac field is decomposed as follows:

$$\psi(x) = \sum_{n=1}^{N} \Big(a_n\, \psi_n^{(+)}(x) + b_n^\dagger\, \psi_n^{(-)}(x) \Big) + \ldots ,$$

$$\{ a_m , a_n^\dagger \} = \{ b_m , b_n^\dagger \} = \delta_{m,n} , \quad a_n|0\rangle = b_n|0\rangle = 0 .$$

Here the positive/negative-frequency (in a sense, see [7]) wave functions $\{\psi_n^{(\pm)}(x)\}$ do not depend on $\{a_n, a_n^\dagger, b_n, b_n^\dagger\}$. It is easy to find the vacuum expectation value of the quantity (39) in the second order [9]:

$$\langle T_{\psi\,\mu\nu}\rangle_0 = \mathrm{Re}\left[i \sum_{n=1}^{N} \overline{\psi}_n^{(-)} \gamma^a e^{(0)}_{a(\mu} D^{(0)}_{\nu)} \psi_n^{(-)} \right] . \quad (40)$$

Now let us take into account that the dynamics of universe is governed by the inflation scenario. In the zeroth order the metric is expressed as

$$\mathrm{d}\, s^{(0)\,2} = \mathrm{d}\, t^2 - a^2(t)\, \mathrm{d}\,\Omega^2 , \quad (41)$$

where $\mathrm{d}\,\Omega^2$ is the metric on unite sphere S^3 and $a(t)$ is the scale factor of universe. It follows from the Dirac equation that the integrals

$$\int \mathrm{d}\, V^{(0)}(t)\, \psi_m^{(\pm)\dagger}(t) \psi_n^{(\pm)}(t) = \delta_{mn} \quad (42)$$

are conserved. Using Eqs. (42) we obtain the estimations for the wave functions in (40):

$$\left| \overline{\psi}_n^{(\pm)}(t)\, \psi_n^{(\pm)}(t) \right| \sim 1/a^3(t) . \quad (43)$$

Therefore the estimation for the value (40) is as follows:

$$\langle T_{\psi\,\mu\nu}(t)\rangle_0 \sim (N\, k_{max})/a^3(t). \quad (44)$$

It is naturally to suppose that $k_{max} \sim l_P^{-1} \sim 10^{33} cm^{-1}$, or $k_{max} \sim k_{SS}$ if supersymmetry is restored at $|\mathbf{k}| > k_{SS} \sim 10^4 GeV \sim 10^{18} cm^{-1}$.

To estimate the number N in (44) (the total number of degrees of freedom) I use the following formula:

$$N \sim \int^{k_{max}} \frac{\mathrm{d}^3\, k}{(\Delta k_{min})^3} \left(\frac{\Delta k_{min}}{|\,\mathbf{k}\,|} \right)^{\alpha} , \quad \alpha = 1. \quad (45)$$

The measure under the integral (45) is Lorentz-invariant and the value Δk_{min} has the sense of the nearest momenta difference modulo. For $\alpha = 0$ and $\Delta k_{min} = 2\pi/a(t)$ the measure in (45) coincides with the usual one for dense mode packing. I take

$$\Delta k_{min} \sim 10^{21} cm \sim 10^{-7} L , \quad (46)$$

where $L = 10^{28} cm$ (the dimension of observed part of Universe).

Thus from Eqs. (44)–(45) an interesting inequality is obtained:

$$16\pi G \langle T_{\mu\nu}\rangle_0 \sim \frac{l_P^2\, k_{max}^3}{(\Delta k_{min})^2 a^3(t_0)} \leq \Lambda . \quad (47)$$

From here the following estimation for the present dimension of universe is find:

$$a(t_0) \geq 10^{15} L . \quad (48)$$

Analogously, in supersymmetric case

$$a(t_0) \geq L\,. \tag{49}$$

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[8] I mean here that the partition function is a functional of the values of the dynamic variables at the boundary $\partial\,\mathfrak{K}$.

[9] The order of approximation means the total degree of operators $\{A_n^{\dagger},\, A_N\}$ taken into account.